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REGRESSION RATES IN BOUNDARY LAYER COMBUSTION

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This paper is concerned with the re-examination of the hybrid regression rate equation proposed by Marxman and co-workers and long accepted in the combustion literature. The important conclusion of the theory that the regression rate, being a weak function of the mass transfer number B, is nearly independent of the properties of the fuel and oxidiser is shown not to be borne out by the experimental studies including those of Marxman et al. Experiments specifically designed by the present authors to elucidate the dependence of the mass transfer number B are described. It is shown that the exponent on B is 0.5 instead of 0.23 from the data on rubber − oxygen + nitrogen system. A theory is proposed here to explain these results. One element of the theory is that the density variation across the boundary layer as affected by temperature and, more importantly, molecular weight has significant effect on the blocking effect (reduction in skin friction coefficient with injection).

It is first shown that the theory explains the long standing results on the blocking effect with denser gas injection. This result is then combined with the known fact that in polymer combustion environment the mean molecular weight of fuel gases near the surface is much larger than monomer molecular weight by a factor of 4–6 to obtain modified predictions on regression rate. A new regression rate law has been obtained incorporating these features and it reads as

$$\rho_p \dot{r} = 0.056\, G^{0.8} (x/\mu)^{-0.2} (\rho_f/\rho_e)^{0.71} (\rho_w/\rho_e)^{0.14} \cdot B\,(1 + B)^{-0.73+0.002\rho_w/\rho_e}$$

where ρ_w, ρ_e and ρ_f are the densities at the central core, wall and the flame respectively. The predicted results are shown to remove the anomalies present in the earlier prediction procedures. The new regression rate law may have far reaching consequences in hybrid regression rate theory as well as erosive combustion in solid propellants.

Introduction

The combustion in a hybrid rocket engine has been studied over the last two decades. These consist of studies in laminar and turbulent flows. In the present paper the studies of turbulent flows only are treated and the work on laminar flows will not be discussed. One of the studies on turbulent flows relevant to hybrid rocket engine, which has considerably enhanced the understanding is that of Marxman et al.[1–4] These authors have identified the hybrid combustion to be similar to that of a turbulent boundary layer over a flat plate with injection of fuel from the surface. The principal results of this study were that the regression rate of hybrid fuel is a strong function of mass flux through the port, is independent of pressure over the range of practical interest of the rocket engines the combustion being diffusion limited, and is a weak function of the properties of fuel and oxidizer. This work has been quoted by many subsequent authors and the basic conclusions have not been seriously questioned.

However, a careful examination of the experimental data on regression rate shows that some of the above conclusions are not borne out. The experiments of Wooldridge et al[9] show that the difference in regression rates among various fuels is much more than that can be predicted by Marxman's theory even considering the uncertainty in evaluating some of the thermophysical properties. Since the effect of properties of the fuel-oxidant combination appears through the mass transfer number B, it is necessary to evaluate the dependence on this parameter experimentally. Further, better understanding of the combustion is needed to explain the observed experimental evidence mentioned above. In this paper an attempt has been made to resolve some of these points.

In the next section, a closer examination of the theory of Marxman and co-workers[1-4,9] is presented and important discrepancies between the theory and experiments brought out. Next some experiments conducted by the present authors to clearly bring out the dependence of regression rate on B are described. A new theory is developed to account for the above observations based on some experimental results on boundary layer with foreign gas injection. The comparison of this new theory (i) with the present experiments and (ii) with those of Wooldridge et al.[9] is made and shown that the observed features can be explained by the new theory.

A Critique of Marxman's Theory

Marxman and co-workers[1-4] developed a theory on hybrid rocket combustion assuming the flow in the combustion chamber to be similar to turbulent boundary layer on a flat plate with fluid injection. Invoking Reynold's analogy and using the results of skin friction for zero injection they obtained an expression for regression rate of hybrid fuels as

$$\rho_p \dot{r} = 0.03\,\mathrm{R}_x^{-0.2}\,\mathrm{G}\,(\mathrm{C}_f/\mathrm{C}_{f_o})\,\mathrm{B} \tag{1}$$

This result delineates the effects of fluid mechanics and thermochemistry. The effect of fluid mechanics is represented by the factor $\mathrm{R}_x^{-0.2}\,\mathrm{G}$ and thermochemistry by $\mathrm{B}\,(\mathrm{C}_f/\mathrm{C}_{f_o})$. The ratio $(\mathrm{C}_f/\mathrm{C}_{f_o})$ represents the reduction in skin friction or heat transfer due to injection (the blocking effect). This term was evaluated by Lees[5] from simple film theory as

$$\mathrm{C}_f/\mathrm{C}_{f_o} = \ln(1+\mathrm{B})/\mathrm{B} \tag{2}$$

Marxman and Gilbert[1] used Eq. (2) to obtain C_f as

$$\mathrm{C}_f = 0.0225\,[\ln(1+\mathrm{B})/\mathrm{B}]\,\mathrm{R}_\delta^{-0.25} \tag{3}$$

Equation (3) is substituted in the momentum integral equation and on integration, an expression for $\mathrm{C}_f/\mathrm{C}_{f_o}$ which when plotted on a log-log plot leads approximately to

$$\mathrm{C}_f/\mathrm{C}_{f_o} = 1.2\,\mathrm{B}^{-0.77} \tag{4}$$

To check this result, Marxman[2] compared it with the experimental results of Mickley and Davies,[6] Pappas and Okuno[7] and Tewfik.[8] The comparison between Eq. (4) and the results of Ref. 6 was shown to be satisfactory.

Substitution of Eq. (4) in (1) leads to the final result

$$\rho_p \dot{r} = 0.036\,\mathrm{G}\,\mathrm{R}_x^{-0.2}\,\mathrm{B}^{0.23} \tag{5}$$

This expression is further modified[2] to account for density variation inside the boundary layer by a simple correlation like

$$\rho_p \dot{r} = 0.036\,\mathrm{G}\,\mathrm{R}_x^{-0.2}\,\mathrm{B}^{0.23}\,(\bar{\rho}/\rho_e)^{0.6} \tag{6}$$

where the ratio of mean to free stream density, $\bar{\rho}/\rho_e$ is given by

$$\left(\frac{\bar{\rho}}{\rho_e}\right)^{0.6} = \frac{(\delta/x)\,\mathrm{R}_x^{0.2}\,[(\rho_b/\rho_e) + 1 + [(\rho_w/\rho_e) - 1)\eta_b]}{2[(0.0281/\mathrm{I})(1+\mathrm{B})\ln(1+\mathrm{B})/\mathrm{B}]^{0.8}} \tag{7}$$

where the notations are the same as in Ref. 2.

The term $(\delta/x)\,\mathrm{R}_x^{0.2}$ in the numerator of Eq. (7) is expected to account for the deviations due to density variation across the boundary layer. It has to be obtained from experiments and it is claimed that it depends on the fuel-oxidant combination only. Its variation with the core temperature (which may be at ambient conditions in some situations or at higher temperature in others) is not known experimentally or otherwise. If a large core temperature (small core density) is used in Eq. (7), by taking a value of $(\delta/x)\,\mathrm{R}_x^{0.2}$ of about 1.25 given in Ref. 2, the density correction factor is about 10, a number which appears unrealistic. Wooldridge et al.[9] use the correction factor $(\bar{\rho}/\rho_e)^{0.6}$ as unity in their later predictions for various fuels used in the experiments. As such the methodology to take account of density variation is not satisfactory.

It can be seen that $\rho_p \dot{r}$, as given by Eq. (5), is relatively insensitive to B due to the low exponent (0.23) on B. This constitutes one of the principal results of the theory stressed by Marxman and co-workers.[2-4] The above theory was compared with the experimental results of Plexiglas-Oxygen system obtained at ambient pressure in two dimensional burners and the comparison was claimed to be satisfactory.[1,2] The estimates for B were made for essentially diffusion limited combustion in the above comparison. The aspect of diffusion limitedness does not affect the basic nature of the expression (6) but only alters the value of B which is influenced by the wall oxidiser concentration. Wooldridge et al.[9] in an elaborate study not often referred to in the open literature have presented extensive and useful data on the regression rate at varying pressures obtained for Plexiglas (PMMA), Polyurethane (PU) and PBAN in a cylindrical rocket motor. Two of these are presented as fuel weight loss vs. burn time data and are shown in Fig. 1.

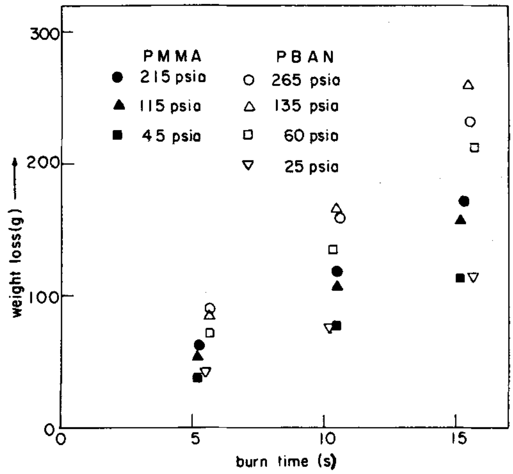

FIG. 1. Results of Wooldridge et al. (Ref. 9) on PBAN and PMMA—O_2 system.

The authors claim good comparison between the theory and experimental results at high pressures (about 215–265 psi). The fuel weight loss at low pressures (~45 psi) is substantially less than at 215 psi. It is to be recalled as noted earlier that good comparison between diffusion limited theory and near ambient pressure experiments was reported in Refs. 1 and 2. Thus one is led to the anomalous situation where the same theory for diffusion limited conditions is said to have compared well with numerically widely different results.

This situation has been carefully analysed by examining Ref. 9. It appears that the mass flux G used in the theoretical estimate goes upto 1.6 times the mean mass flux. The motivation for the use of increased flux comes from the fact that in the relation for $\rho_p \dot{r}$ derived from boundary layer analysis, G is the free stream flux and in an enclosed flow, by analogy, maximum flux rather than the average flux should be used. But it should be pointed out that the maximum flux in a fully developed pipe flow is only about 20 percent more than the average flux. The factor used in the computer analysis of data by Wooldridge et al.[9] increases to 60 percent in a distance of about 5 times the port diameter. This distance is based on the fact that the boundary layer growing on the fuel wall will merge at about five times the port diameter. In actuality, the length for merger of boundary layers depends on the inlet Reynolds number. In view of the above, the claim of good comparison is not indeed justified. To further establish the lacuna of this theory it is worth examining the results of Ref. 9 summarised in Table I. It can be observed that the predictions from Marxman's theory and the experimental results are widely different. Further, comparison of results for PBAN and PU with nearly identical B and at same flux indicates a difference in mass regression rate of nearly 18 percent. Even if it is stated that the estimate of B needs modification, to explain this variation in regression rate, the values of B for PBAN—O_2 and PU—O_2 would need to be different by a factor 2. This is too large a factor to be expected.

A further confirmation of the possible influence of B comes from the effect of pressure (Fig. 1). It is seen that the regression rate decreases by a factor 0.7 for PMMA—O_2 and 0.5 for PBAN—O_2 when pressure is decreased from 215 psia to 45 psia and 265 psia to 25 psia respectively. Since pressure effects have to enter through kinetics—largely of gas phase nature in these non-hypergolic systems—their influence should be felt through B only. Again one can estimate the variation in B needed to account for the reduction of regression rate of these magnitudes. They turn out to be 4.7 and 20 (with the exponent of 0.23 on B as in the theory). These factors on variation in B are too unrealistic.

Thus a careful examination of the experimental work of Marxman and co-workers suggests that the law needs strong corrections particularly with the thermochemical factor. A further evidence about it can be obtained by examining the relative regres-

TABLE I
Weight loss data of fuel-oxygen system of Wooldridge et al. (Ref. 9) and present theoretical results

Fuel	B (Ref. 9)	G_{ox}, in† g/cm^2 s	Observed fuel weight loss (in g)	Fuel weight loss estimated by Marxman's theory (in g)	Molecular weight of gases near wall	Weight loss by present theory, g
PMMA	9.3	7.03	160	130	90–100	180–190
PBAN	8.0	7.73	260	132	250–350	260–280
PU	8.0	7.73	220	132	150–250	220–250

†Initial oxidiser mass flux.
Experiments in diffusion-limited conditions with a 1″ ID, 12″ long motor.

sion rates of several fuels by two other investigators—Blazowski et al.[10] and Joulain et al..[11] The ordering of the fuels in terms of regression rate with oxygen is shown below:

Blazowski et al.[10] : Delrin ≥ PMMA ≥ PP > PE > PSt

Joulain et al.[11] : PSt > PMMA > Delrin > PP > PE

(PP = Polypropylene, PSt = Polystyrene, PE = Polyethylene)

The experiments of Blazowski et al. are in stagnation point flow apparatus and that of Joulain et al. in a two dimensional flat plate burner; but the nature of apparatus should not matter since thermochemical influence will be similar in both cases. The point to be noticed in the above ordering is that even qualitatively, fuels are classed differently by different workers. The mass regression rate ratio between extreme fuels is about 1.6 for Ref. 10 and 2.0 for Ref. 11. The results of Joulain et al. with regard to polystyrene and PMMA correspond to those observed by other workers (Houser and Peck[12] and Srivastava[13]). It appears from these that even though fuels with same name may have been used by different workers, they would probably have antioxidants and other additives which could affect the regression rate. It is the objective of the present work to elucidate the features responsible for the above experimental observations.

The Present Experiments

An examination of earlier literature reveals that no experiments have specifically been carried out to elucidate the dependence of regression rate on B. Most researchers have used different polymeric fuels and it is indeed difficult to estimate changes in B from fuel to fuel as the factor h_v in $B = \Delta h_g / h_v$ is composed of relatively less certain heat of degradation to a fragment size larger than the basic monomer. Therefore it was thought worth while conducting experiments to estimate the effect of varying B on regression rate by varying $(\Delta h)_g$. This is done by varying the gaseous composition (of say an oxygen + nitrogen mixture).

The apparatus used in the present experiments is conventional. Two separate sources of high pressure gaseous oxygen and nitrogen are connected to a cylindrical motor through regulators, choked orifices and a mixing chamber. The fuel blocks are composed of natural rubber (poly isoprene) and a special fuel, difurfurylidiene cyclo hexanone (DFCH), in the ratio 1:3 by weight.[14] The fuel blocks are cylindrical with 20 mm ID × 40 mm OD × 160–240 mm long. Several identical motor configurations were test fired for different durations and the chamber pressure is tuned to a level which is otherwise established in a separate series of experiments to lead to diffusion limited combustion. The chamber pressure was set to 12 atm. The weight loss of the fuel, the chamber pressure, the flow rates of the gaseous oxygen and nitrogen as monitored by the upstream pressure transducers, and burn duration are the results of the experiments. Most of the data are recorded on an analog instrumentation tape recorder and subsequently replayed for analysis.

Analysis of Data

The objective of the analysis of data is to distinguish between effects of B and the oxidiser flow rate on regression. The regression rate expression can be written as

$$\rho_p \dot{r} = a\, G^n x^{n-1} \tag{8}$$

where a is a function of B. The value of a for various levels of dilution of oxygen used in the test is to be evaluated. Equation (9) can be integrated over x the axial distance and time to obtain

$$\int_{D_o}^{D_f} \frac{D\, dD}{[1 + KD^{1-2n}]} = \frac{2\, \dot{m}_{ox}\, t_b}{\pi L\, \rho_p} \tag{9}$$

where

$$K = (1 - n)\, 4^n\, a\, L^n\, \dot{m}_{ox}^{n-1} / [n\, \pi^{n-1}] \tag{10}$$

The derivation of Eq. (9) is described in Paul et al.[14] D_o and D_f refer to the initial and final diameters of the fuel grain and L is the length of the fuel grain. The value of D_f is calculated from weight loss measurements. All parameters in the Eq. (9) and (10) are known excepting a. In the present data analysis n is taken as 0.8 consistent with Eq. (5). It is evaluated by Newton-Raphson procedure for each experiment. In order to compare the results from the present procedure with the predictions of Marxman's theory, the value of a is evaluated using

$$a = 0.036\, B^{0.23}\, \mu^{0.2} \tag{11}$$

which is obtained from Eq. (5) and (8). The value of viscosity is taken equal to 1.8×10^{-4} g/(cm)(s) a figure quoted by Wooldridge et al. in Ref. 9. B is obtained from

$$B = \frac{c_p\, (T_{in} - T_w) + H\, Y_{o\,in}/i}{h_v}$$

where T_{in} and $Y_{o\,in}$ refer to temperature and oxidiser concentration of the input stream. The values for various parameters are chosen as below:

$$c_p = 0.4 \text{ cal/gm° C}, \quad T_{in} = 300° \text{ K}, \quad i = 2.5$$

(taken for oxidation to CO_2 and H_2O), H = 7000 cal/gm fuel.

The effective heat of degradation was obtained from experiments on differential scanning calorimetry and other data, to be about 175 cal/g.[25]

Experimental Results

Figure 2 shows the plot of weight loss vs burn time for various chamber pressures. It is seen clearly that in the pressure range of 7–18 atm the weight loss is independent of pressure showing that diffusion limitedness has been achieved. The results of analysis of the experimental data on a vs B are shown in Fig. 3. It is seen that the data points show a magnitude of a much larger than Marxman's and though the data scatter is not small, the slope of the data trend is larger than Marxman's. A curve fit of the data leads to $a \sim B.^{0.51}$ Thus, the present experiments seem to suggest dependance on B much stronger than in Marxman's theory.

Theoretical—Preliminary Observations

While an examination of Eq. (2) and the above data on the exponent of B indicate that the cause of the difference may be in the estimation of the blocking effect, the other possible causes like charring and melting have been considered as possibilities. The char layer acts as a barrier to the heat and mass transfer between the flame and the pyrolysing surface and reduces the regression rate. However, this cannot explain the anomalies described in the paper. In the experiments of Wooldridge et al,[9] only PBAN showed traces of charring; yet it had the maximum regression rate. The difference in regression rate between PU and PMMA (both of which do not produce char layer) will remain unexplained. Non-charring fuel blocks were selected for the present experiments and moreover, the mass transfer number B was varied by varying the concentration of oxygen in inlet stream. For these reasons, the anomalies described in the paper cannot be attributed to surface charring.

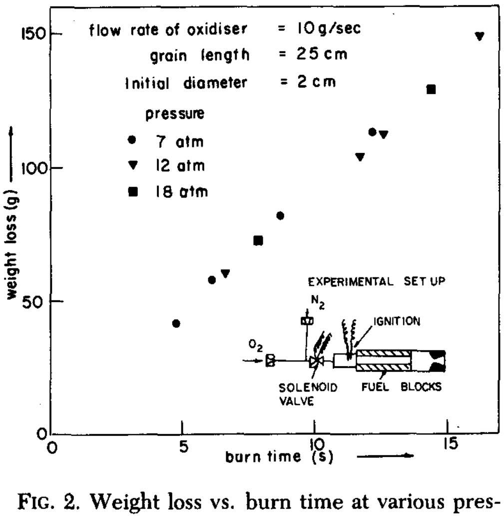

FIG. 2. Weight loss vs. burn time at various pressures from present measurements.

Further melting may also affect the regression rate if sufficiently thick layer is present at the surface. In all the experiments described in the paper, the regression rate is too large to produce a thick melt layer in any of the fuels. The thermal layer thickness in the fuels in the range of regression rate estimated is of the order of 200 to 300 μ. Out of this the melt layer thickness could at best be about 100 μ and the residence time of the melt layer is about 0.25 s. Under these conditions melting is not likely to affect the results much.

The effect of combustion to CO or CO_2 must be felt through the chemical kinetic effects, otherwise calculations assuming equilibrium at the flame must be satisfactory. As has been pointed out earlier, the effect of gas phase kinetics on regression rate is felt through the concentration of oxidizer at the surface and this is shown to be small in the operating regime of the rocket engines. Moreover, the results compared are in the regime independent of pressure and hence kinetic effects can be ignored.

The concept of partial oxidation to CO has been used in estimating B by Marxman et al.[3] The change in B due to this will be not sufficient to explain differences in regression rates, more particularly when different fuels are considered. All of these happen because of the low exponent on B. Consequently the only point that needs examination is the blocking effect which is what is considered now.

While examining the literature[16,17] on C_f/C_{f_o} it became clear that the blocking effect is a function of the molecular weight of the injected species (M_w). In fact, the blocking effect decreased with increase in M_w. In typical hybrid situations (see Rabinovich[23] and Kumar and Stickler[24]) the molecular weight of the species at the wall is 100–400 or more whereas the molecular weight of the free stream composed primarily of oxidising species and diffused products, will be about 24–28. The large difference in molecular weights between wall and free stream can cause a significant reduction in

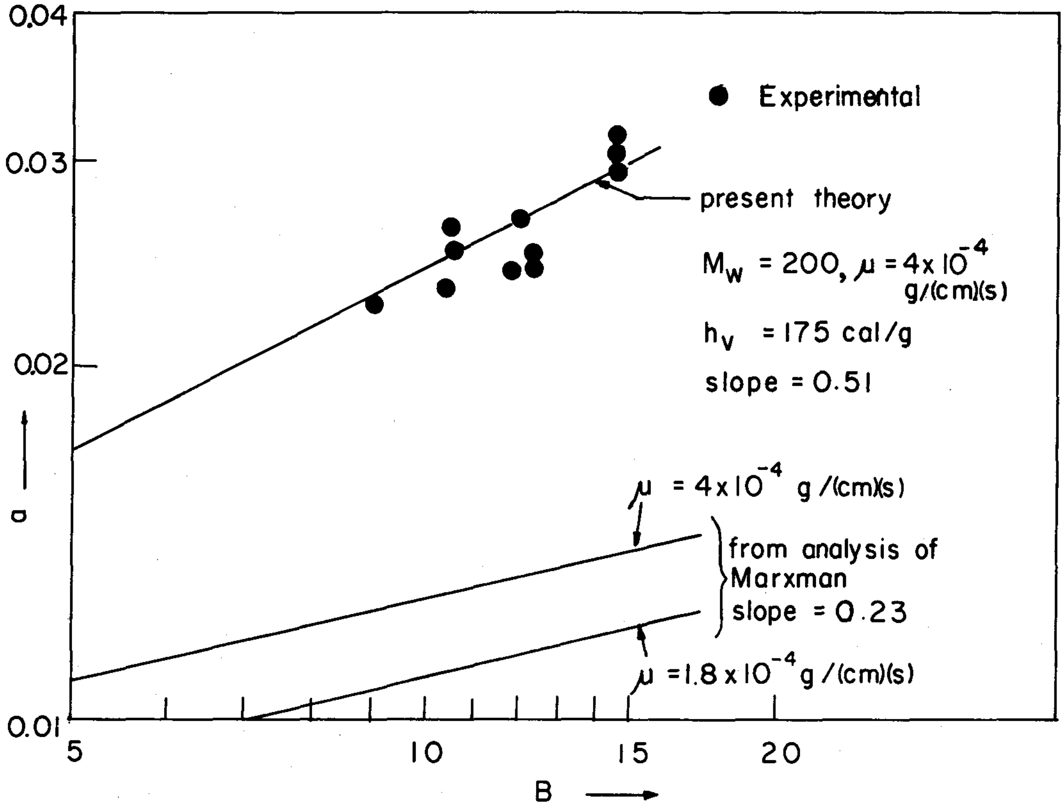

FIG. 3. Results of a vs. B from present experiments and theory as well as Marxman's theory.

blocking effect, a feature not incorporated in Marxman's theory.

Another factor is associated with the temperature profile in the region close to the surface. Measured temperature profiles (at ambient pressure) of Wooldridge and Muzzy[18] for H_2—(O_2 + N_2) hybrid system show a peak at the flame and a drop off to much lower values on the free stream side. This situation will possibly not be obtained in a rocket motor operating at higher pressures like in the experiments of Wooldridge et al..[9] The measured temperature profile by Helman and Timnat[19] in a hybrid rocket motor by optical techniques at a distance not far from the leading edge shows a core temperature close to adiabatic flame temperature rather than near ambient temperature. Further, in the experiments by the present authors, there was indirect evidence of high core temperature even slightly ahead of the leading edge of the fuel blocks. The regression rate apparatus described earlier used stainless steel probes for connecting the bridge wire which ignited solid propellant powder. The probes kept ahead of the fuel blocks (see inset in Fig. 2) melted in most tests with burn duration more than 6 or 7 seconds. There would be no reason for melting of the probes (melting point being about 1500° C) unless the core is at high temperature. Thus there seems to be reasonable evidence to believe that the temperature profiles at higher pressures are different from the classically assumed profiles of the kind measured by Wooldridge and Muzzy.[18]

Thus it is necessary to evaluate the effects of density variation as affected by temperature and molecular weight of the species. This is attempted next.

The Theory

a) *Blocking Effect*

The velocity profile for the inner layer of the boundary layer with injection can be derived by assuming the one dimensional form of momentum equation as[5]

$$\tau = \tau_w + \rho u v \qquad (12)$$

The shear stress, τ is evaluated from Prandtl's mixing length theory as

$$\tau = k^2 y^2 (\partial u/\partial y)^2 \tag{13}$$

where k is a constant (0.4). Substituting expression (13) into Eq. (12) and rearranging it, one gets

$$\frac{d(u/u_\tau)}{\sqrt{\dfrac{\rho}{\rho_e} + \dfrac{u}{u_\tau}\dfrac{v}{u_\tau}}} = \frac{d(yu_\tau/\nu)}{k\left(\dfrac{yu_\tau}{\nu}\right)} \tag{14}$$

where u_τ is the friction velocity $(=\sqrt{\tau_w/\rho})$. Equation (14) rewritten in terms of B leads to

$$\frac{1}{u_\tau}\left[\frac{\rho}{\rho_e}\frac{1}{1+\mathrm{B}u/u_e}\right]^{1/2} d(u/u_e) = \frac{d(yu_\tau/\nu)}{k(yu_\tau/\nu)} \tag{15}$$

Integration of Eq. (14) yields the law of wall for the boundary layer as

$$\psi^+ \equiv \int_o^{u/u_\tau} \frac{d\zeta}{\left[\dfrac{\rho_e}{\rho}\left\{1 + \dfrac{\rho v}{\rho_e u_\tau}\right\}\right]^{1/2}} = \frac{1}{k}\ln\frac{yu_\tau}{\nu} + \mathrm{C} \tag{16}$$

Essentially the same form of equation has been derived by many authors for the velocity profile in the boundary layer with injection. Stevenson[19] derived the wall law for constant density flow in which case integration of the left hand side of Eq. (16) is straightforward. Equation (16) is in fact similar to the wall law without injection, the only difference being the replacement of u^+ $(=u/u_\tau)$ by ψ^+ in the present case. Therefore the velocity profile of the outer layer in the overlap region can be written as

$$(\psi^+ - \psi_e^+) = \frac{1}{k}\ln\left(\frac{y}{\delta}\right) - \mathrm{D} \tag{17}$$

Subtracting Eq. (17) from Eq. (16), one gets

$$\psi_e^+ = \frac{1}{k}\ln\frac{\delta u_\tau}{\nu} + (\mathrm{C} + \mathrm{D}) \tag{18}$$

where ψ_e^+ is the value of ψ^+ at the edge of the boundary layer.

Now it is assumed that the constants C and D do not change with injection. This assumption has earlier been used successfully by Boccio and Economos[21] in an integral method for computing turbulent boundary layer with injection and constant density. Using the values for C and D from flow without injection and rearranging Eq. (18), one obtains

$$\frac{\varphi}{(\mathrm{C}_f/2)^{1/2}} = \frac{1}{k}\ln\left(\frac{\mathrm{C}_f}{2}\mathrm{R}_\delta\right) + 8 \tag{19}$$

where

$$\varphi = \int_o^1 \left\{\frac{\rho}{\rho_e}\frac{1}{1+\mathrm{B}\zeta}\right\}^{1/2} d\zeta \tag{20}$$

Equation (20) can be used for evaluating C_f. The reduction in C_f due to blowing and any modification due to density gradients are included in the factor φ. If it is assumed that the right hand side of Eq. (20) remains unchanged by injection—this will be largely true since in the right hand side $(1/k \ln \mathrm{R}_\delta + 8)$ is large compared to $1/k \ln (\mathrm{C}_f/2)$—one obtains

$$\frac{\mathrm{C}_f}{\mathrm{C}_{fo}} = \varphi^2 \tag{21}$$

In order to evaluate φ in Eq. (20), the density profile across the boundary layer has to be defined. When there is no combustion this can be evaluated by assuming similarity between the momentum and mass transfer. The concentration of the injected species then becomes a linear function of velocity, and we obtain

$$\frac{\rho}{\rho_e} = \left[\left\{1 - \frac{\mathrm{B}}{1+\mathrm{B}}\rho_{ei}\right\}\frac{u}{u_e} + \frac{1+\mathrm{B}\,\rho_{ei}}{1+\mathrm{B}}\right]^{-1} \tag{22}$$

where ρ_{ei} is the ratio of the density in the free stream to that of the injected species, ρ_e/ρ_i. Substitution of Eq. (22) into Eq. (21) yields

$$\varphi = \left(\frac{\mathrm{C}_f}{\mathrm{C}_{fo}}\right)^{1/2} = 2\left\{\frac{1+\mathrm{B}}{\mathrm{B}^2(1-\rho_{ei})}\right\}^{1/2} \cdot \ln\left\{\frac{(1+\mathrm{B})^{1/2}(1+\sqrt{1-\rho_{ei}})}{\sqrt{1+\mathrm{B}_{ei}}+\sqrt{1-\rho_{ei}}}\right\} \tag{23}$$

A comparison between the predictions of this equation and the experimental results of Romanenko and Kharehenko[16] on the injection of CO_2 and freon into air stream is shown in Fig. 4. As can be noticed the agreement is excellent.

When combustion is present, the problem is solved using the following procedure. The variation of density ρ_e/ρ is assumed to have linear relation with the velocity (u/u_e) in the regions above and below the flame, each taken separately. The expression for φ can then be obtained as

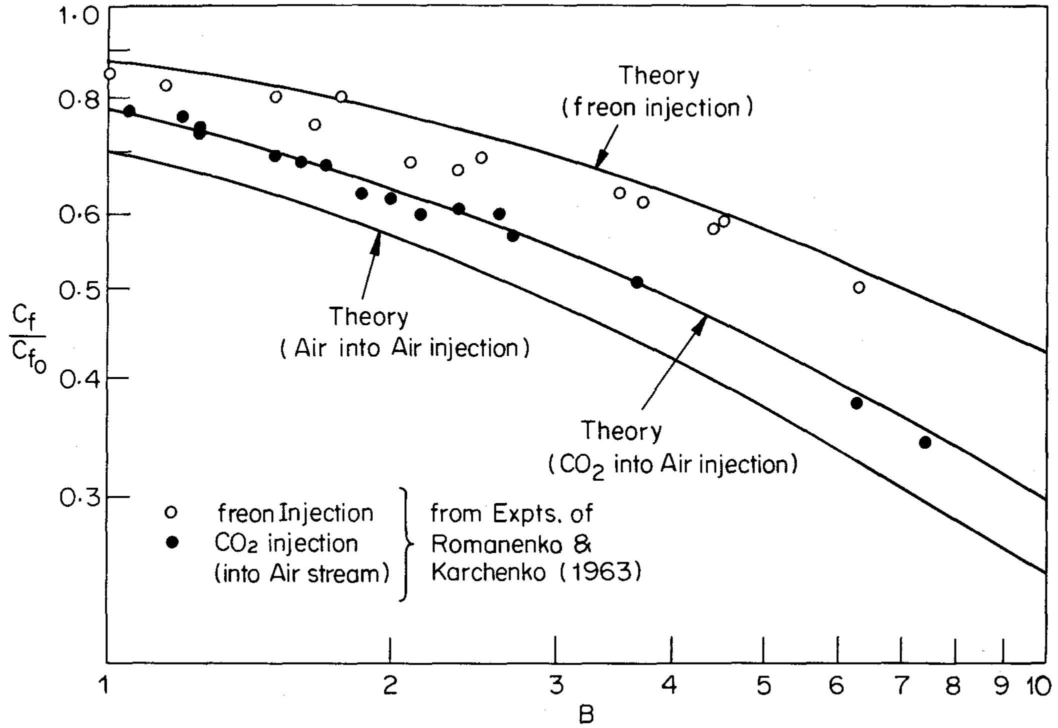

FIG. 4. Comparison between experiments of Romanenko and Kharchenko (Ref. 16) and predictions from present theory.

$$\varphi = \frac{u_f}{u_e} I_1 + \left(1 - \frac{u_f}{u_e}\right) I_2 \qquad (24)$$

where u_f is the axial velocity at the flame and I_1, I_2 are integrals over the boundary layer which depend on the flame position and the densities at the wall, at the flame and in the free stream. Noting that at the flame $(Y_1 - Y_o/i)$ becomes zero and assuming that similarity exists between mass and momentum transfer, the expression for u_f/u_e can be obtained as

$$\frac{u_f}{u_e} = (B - Y_{oe}/i)/[B(1 + Y_{oe}/i)] \qquad (25)$$

Details of calculation of the integrals I_1 and I_2 are available in Ref. 25.

The value of φ given by Eq. (24) is used to evaluate the blocking effect in the presence of combustion.

b) *Regression Rate*

We begin by rewriting Eq. (1)

$$\rho_p \dot{r} = G \frac{C_f}{2} B. \qquad (26)$$

Now it is possible to obtain C_f if R_δ can be determined, say from the momentum integral equation. Alternatively the velocity profile of Eq. (16) can be used in the momentum integral equation along with an appropriate wake function as done by Boccio and Economus.[20] As neither of these procedures gives explicit relationships for the regression rate, the following procedure is adopted. The stanton number C_h is expressed by the empirical relation[22]

$$C_h = 0.015\, R_h^{-0.25} \left(\frac{C_h}{C_{ho}}\right) \qquad (27)$$

The enthalpy integral equation of the boundary layer is

$$\frac{d\delta_h}{dx} = C_h (1 + B) \qquad (28)$$

Solution of Eq. (28) after substituting for C_h from Eq. (27) can be cast as

$$R_h = 0.0427 \left(\frac{C_h}{C_{ho}}\right)^{0.8} R_x^{0.8} (1 + B)^{0.8} \qquad (29)$$

The final expression for regression rate can then be written as

$$\rho_p \dot{r} = 0.033\, G^{0.8} (x/\mu)^{-0.2} \cdot (C_h/C_{ho})^{0.8}\, B/(1+B)^{0.2} \quad (30)$$

In the above expression C_h/C_{ho} ($=C_f/C_{fo}$ by Reynolds analogy) is obtained by procedure described earlier.

Since the calculation of C_f/C_{fo} involves evaluation of rather complicated expression, a simplified approximate relation was obtained by curve fitting the values of C_f/C_{fo} calculated for various values of ρ_f/ρ_e, ρ_w/ρ_e and B. The final relation for regression rate is obtained as

$$\rho_p \dot{r} = 0.056\, G^{0.8} (x/\mu)^{-0.2} (\rho_f/\rho_e)^{0.71} (\rho_w/\rho_e)^{0.14} \cdot B\,(1+B)^{-(0.73-0.002\,\rho_w/\rho_e)} \quad (31)$$

Equation (31) fits Eq. (30) within an accuracy of ±5 percent in the range of interest to hybrid rocket operation. ($\rho_f/\rho_e \sim 0.5$–1, $\rho_w/\rho_e \sim 5$–30, $B \sim 5$–50). There are also other factors like flame position (which is a function of stoichiometric ratio) which affect the result; but since their effect is small they are not considered while developing Eq. (31). The nature of dependence of regression rate on various parameters can be seen in Eq. (31); but the dependence of regression rate on B is stronger than what is apparent from the equation since the density ratio ρ_w/ρ_e depends on B as well.

Effect of the Core Temperature

It is interesting to determine the effect of varying the core temperature on the blocking effect. Computations have been made using the procedure detailed earlier for a case when the flame temperature is 2500 K. The results are shown in Fig. 5.

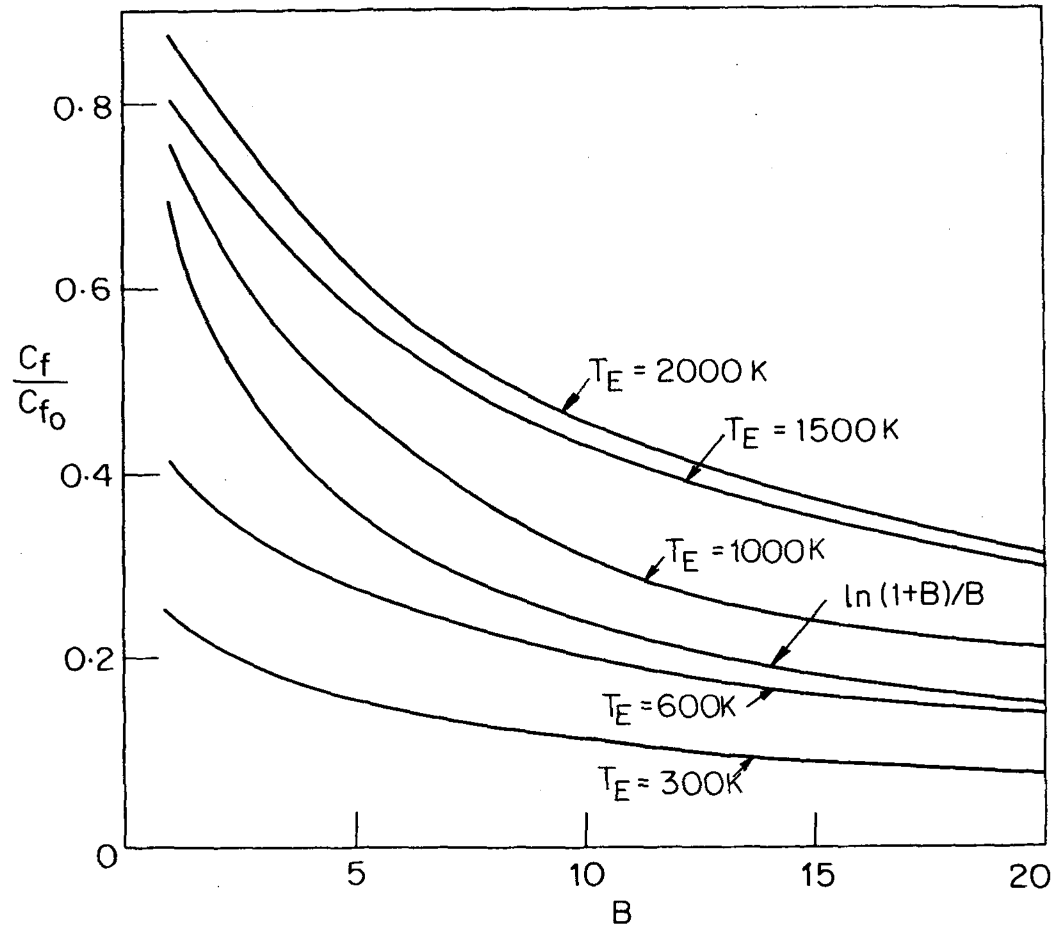

FIG. 5. Effect of core temperature on blocking effect.

It is seen that at a typical value for B of 10, increase in core temperature from 300 to 2000 K increases C_f/C_{f_0} nearly four fold (from 0.12 to 0.44 and hence also the regression rate.

Predictions for a Rocket Motor

The above theory is compared with the data of (i) Wooldridge et al.[9] and (ii) the present experiments.

The polymers considered by Wooldridge et al.[9] are PMMA, PBAN and PU. While the flame temperature can be obtained from equilibrium calculations, the molecular weight of fuel vapours near the surface is not easy to obtain. It is known that, when PMMA decomposes slowly, it does so to the monomer. And the typical value of fuel fraction at the surface is about 85 to 90 percent. Combining these facts, it appears reasonable to assume the wall gas molecular weight to be between 90 and 100. The molecular structures of PBAN and PU are more complicated. Following Rabinovich,[23] and Kumar and Stickler[24] it appears that the fragment size near the surface will be much larger than of the monomer. Values over a broad range between 250 and 400 are chosen (Table I) since the numbers are not known accurately. Based on these data the weight-loss has been calculated for the specified burn time for the test conditions given in Table I. The results presented in the last column of Table I can be seen to be in better agreement compared to Marxman's theory.

The calculations for the present experiments have been made taking the average molecular weight of the vaporising fuel to be 200. This value has been chosen since the fuel is 75 percent DFCH whose molecular weight is 254. Some amount of decomposition of the material is likely as it was observed during slow heating of the substance, and the molecular weight of the vaporising fragments of rubber is likely to be in the range of 200–300. The value for h_v is taken as 175 cal/g as already stated. The results of a vs. B from the present theory appear in Fig. 3. The curve of a vs. B has a slope of 0.51. The predicted curve fits the experimental points well both in terms of slope and magnitude.

Pressure Dependence

The data of Wooldridge et al.[9] shown in Fig. 1 indicates substantial reduction in regression rate with decrease in pressure. The general understanding is that the combustion becomes kinetically controlled when pressure is reduced. The effect of kinetics is felt through the reduction in B caused by the increased concentration of oxidiser at the surface due to its incomplete consumption at the flame. To produce regression rate reduction by half the value of oxidiser mass fraction at the surface must increase from zero to 20 percent. The measurements of Wooldridge and Muzzy[18] show a wall oxidiser concentration of 0.3 percent. Therefore it is unlikely that high wall oxidiser concentrations are obtained and so kinetic effect cannot explain the observed pressure dependence.

The calculations with the present theory show that reduction in temperature ratio T_e/T_f from 1 to 0.4 reduces the regression rate by about half. Thus the variation of regression rate with pressure may be explained through variation of core temperature with pressure,[25] a fact observed in the experiments.[2,19] This evidence is not conclusive, but indicative of the possible explanation for the pressure dependence.

Conclusions

Experimental and analytical results on the regression rate of fuels when there is a lateral flow of oxidiser-rich hot gases is first carefully reviewed. It is shown explicitly that application of the presently accepted theory due to Marxman and coworkers to their own experimental results leads to contradictory trends in the comparison of predictions with observed results. The primary aspect considered is the inability of Marxman's theory to explain the 'significant' differences in regression rate amongst various polymeric fuels. The cause of the difference is traced to the estimation of the blocking effect. Using evidence from earlier literature on the reduction in the blocking effect due to injection of denser gas from the wall, a new theory is constructed; this theory revolves around treating density variations in the boundary layer properly. The modified theory is shown to explain (i) the blocking effect in cold flow conditions with heavy gas injection, studied by earlier workers (ii) the observed trends in the regression rate of hybrids. The new regression rate law evolved is

$$\rho_p \dot{r} = 0.056\, G^{0.8} \left(\frac{x}{\mu}\right)^{-0.2} (\rho_f/\rho_e)^{0.71} (\rho_w/\rho_e)^{0.14} \cdot B\,(1 + B)^{-(0.73-0.002\,\rho_w/\rho_e)}$$

In this relation the thermochemical parameters enter through B. Regression rate depends not only on B and mass flux (as in Marxman's theory), but on density ratios also. Consequently the exponent on B which may be observed in an experimental plot will not be constant but varying depending on density ratios.

Nomenclature

a	Constant in regression rate law (g/cm^2 s (g/cm^2 s)^n)
B	Transfer number = $(\Delta h)_g/h_v$
C_f, C_{fo}	Skin friction coefficient with and without injection
C_h, C_{ho}	Stanton numbers with and without injection
G	Mass flux through the port (g/cm^2 s)
h_v	Effective heat of vaporisation of the fuel (cal/g)
$(\Delta h)_g$	Enthalpy difference between free stream and wall (cal/g)
M_w	Molecular weight at the wall
i	Stoichiometric ratio
$\dot{q}''$	Gas phase heat flux to wall (cal/cm^2 s)
$\rho_p \dot{r}$	Mass regression rate (mass flux of fuel) (g/cm^2 s)
R_x, R_δ, R_h	Reynolds numbers based on axial distance, boundary layer thickness and enthalpy thickness
u, v	Axial and normal velocities, along x and y axes (m/s)
u	Friction velocity = $\sqrt{\tau_w/\rho}$ (m/s)
x, y	Axial and normal coordinates
Y	Mass fraction
ρ	Density (g/cm^3)
ρ_p	Solid fuel density (g/cm^3)
τ, τ_w	Shear stress and that at wall
h	Enthalpy thickness (m)
ψ^+	Integral defined in Eq. (17)
φ	Integral defined through Eq. (20)

Subscripts

e,f,w,i	edge of boundary layer/free stream, flame, wall, injectant
p	Solid phase
in	Input stream
o	Oxidiser
l	Fuel

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The authors wish to thank Mr. H. K. Narahari for his assistance in conducting the experiments and Prof. R. Narasimha and Dr. B. N. Raghunandan for reading through the manuscript and giving many useful suggestions.

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COMMENTS

Dr. M. K. King, Atlantic Research Corporation, Alexandria, VA. I would like to point out to the author that the effects of transpiration on skin friction factor and heat transfer coefficient (effects referred to by the author as blocking effects) are treated in considerable detail by Kutateladze and Leont'ev.[1] Included in this text is treatment of the effects of differences in molecular weights of the crossflow and transpiring streams. As shown in Figure 20 of this reference, high molecular weight in the transpiring stream leads to considerable reduction of the blocking effect (increase in transfer coefficient) consistent with the current paper. I would suggest that the authors consider applying the techniques presented by Kutateladze and Leont'ev to analysis of their data.

REFERENCE

1. Kutateladze, S. S., and Leont'ev, A. I., *Turbulent Boundary Layers in Compressible Gases,* Translated by D. B. Spalding, Academic Press, Inc., N.Y., 1964.

Author's Reply. Kutateladze and Leont'ev have presented analysis of cold flow with foreign gas injection. The present analysis is similar to the above work though we have used more accurate expression [Eq. (22)] for the density profile as has also been noted by Spalding as a foot note in the above reference. When combustion is present, density variation is caused by temperature as well as molecular weight variation. The present work is primarily concerned with the above extension to combusting flows. This, it may be noted, has not been treated by earlier workers.

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Dr. M. A. Delichatsios, Factory Mutual Research, Norwood, MA. Marxman's empirical correlation is valid only for convective heat transfer. In some of the fuel slabs you have compared, flame radiation may be the dominant mechanism. Especially, flame radiation dominates at the high pressure tests you used to justify your arguments concerning the B-dependence of the regression rate. In addition the regression rate depends on the flame temperature which varies with the fuel. It is not obvious that you have considered flame temperature variations in your model.

Author's Reply. The effect of radiation has already been included in sufficient detail as seen in Wooldridge and Muzzy! It can be seen from Fig. 3 and 4 of the above reference that, for the range of fluxes of interest to the hybrid rocket engines (~7 g/cm^2 s considered here) radiation has insignificant effect on the regression rate even at pressure as high as 700 psi. For this reason, radiant flux has not been considered in the present analysis. Secondly, the flame temperature variations among different fuels are treated automatically in evaluating B. The best judgment at present is that the results of Wooldridge et al (Ref. 9 of the paper) are not explained on the basis of radiation.

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Dr. M. A. Delichatsios, Factory Mutual Research, USA. To accomplish a closure in your methodology, you postulated a linear relationship of the temperature ratio with the velocity ratio. For combustion such a relationship cannot be conceptually justified, because enthalpy is not conserved due to combustion. A Zhvab-Zeldovich variable should be used to obtain the required closure.

Author's Reply. The existence of similarity between temperature (or sensible enthalpy) and velocity profiles has been demonstrated by experiments of Wooldridge and Muzzy (Ref. 18). Fig. 10, 11 and 12 of Ref. 18 show that the similarity exists at values of B ranging from 2.2 to 54. However, in contrast to Zhvab-Zeldovich variables, similarity for sensible enthalpy exists only if the regions below and above the flame are taken separately. In Ref. 18, sensible enthalpy is nondimensionalised using different expressions (Eqs. 8 and 9 of Ref. 18) for regions below and above the flame. It is only after recognizing this fact that Eq. (20) of the paper is integrated in two steps.